Exercice cinématique du solide

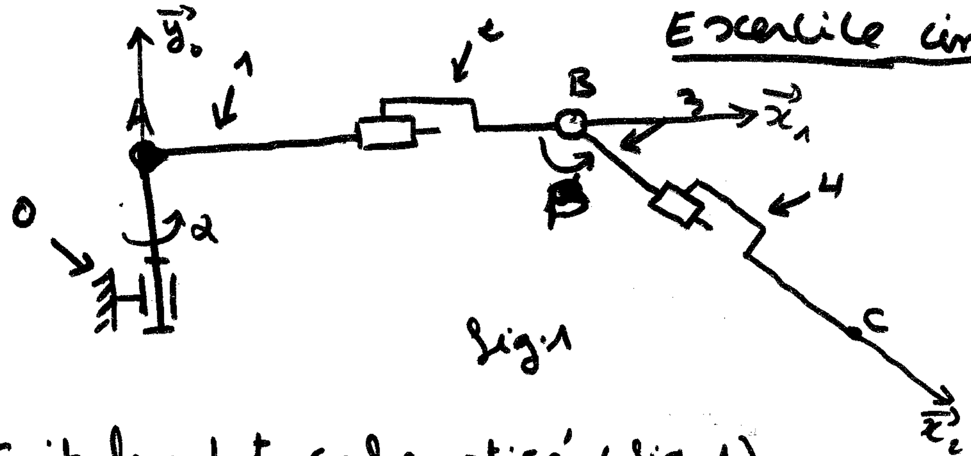


fig. 1

Soit le robot schematisé (fig. 1).

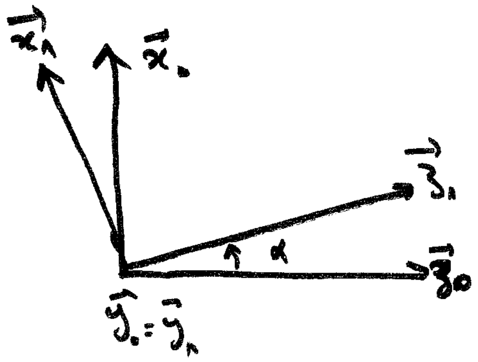
données géométriques :

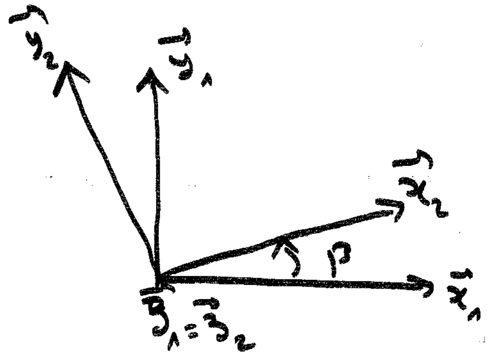
$$\vec{AB} = \lambda_1 \vec{x}_1 \quad ; \quad \vec{BC} = \lambda_1 \vec{x}_2 \quad , \text{tq. } \lambda_1, \lambda_2 \text{ Variables.}$$

on associe au bati un repère $(A, \vec{x}_0, \vec{y}_0, \vec{z}_0)$.

- " aux pièces 1 et 2 un repère $(A, \vec{x}_1, \vec{y}_1, \vec{z}_1)$ tournant autour de R_0 suivant l'axe $(A, \vec{y}_0)$ de l'angle α. $(\widehat{\vec{x}_1, \vec{x}_0}) = (\widehat{\vec{z}_1, \vec{z}_0}) = \alpha$.

- On associe aux pièces 3 et 4 un repère $(B, \vec{x}_2, \vec{y}_2, \vec{z}_2)$ tournant autour de R_0 suivant l'axe $(B, \vec{z}_1)$ de l'angle β. $(\widehat{\vec{x}_2, \vec{x}_1}) = (\widehat{\vec{y}_2, \vec{y}_1}) = \beta$.





Questions :

1 – calculer $\vec{V}(B \in 2/0)$ et $\vec{V}(C \in 4/0)$

2 – calculer $\vec{\Gamma}(B \in 2/0)$ et $\vec{\Gamma}(C \in 4/0)$.

$$-\vec{\Gamma}(C\in 4/0)=\frac{d}{dt/_0}\vec{V}(C\in 4/0)$$

$$=\frac{d}{dt/_0}\left(\dot\lambda_1\vec{x}_1-(\lambda_1\dot\alpha+\lambda_2\dot\alpha\cos\beta)\vec{z}_1+\dot\lambda_2\vec{x}_2+\lambda_2\dot\beta\vec{y}_2\right)$$

$$=\ddot\lambda_1\vec{x}_1-\dot\lambda_1\dot\alpha\vec{z}_1-(\dot\lambda_1\dot\alpha+\lambda_1\ddot\alpha+\dot\lambda_2\dot\alpha\cos\beta-\lambda_2\dot\alpha\dot\beta\sin\beta+\lambda_2\ddot\alpha\cos\beta)\vec{z}_1$$

$$+(\lambda_1\dot\alpha+\lambda_2\dot\alpha\cos\beta)\dot\alpha\vec{x}_1+\ddot\lambda_2\vec{x}_2+\dot\lambda_2(\dot\beta\vec{y}_2-\dot\alpha\cos\beta\vec{z}_2)+\dot\lambda_2\dot\beta\vec{y}_2$$

$$+\lambda_2\ddot\beta\vec{y}_2+\lambda_2\dot\beta(\dot\alpha\sin\beta\vec{z}_2-\dot\beta\vec{x}_2)$$

$$=(\ddot\lambda_1+\lambda_1\dot\alpha^2+\lambda_2\dot\alpha\cos\beta)\vec{x}_1-\Big[2\dot\lambda_1\dot\alpha+2\dot\lambda_2\dot\alpha\cos\beta+(\lambda_1+\lambda_2\cos\beta)\ddot\alpha$$

$$-\lambda_2\dot\alpha\dot\beta\sin\beta\Big]\vec{z}_1+(\ddot\lambda_2-\lambda_2\dot\beta^2)\vec{x}_2+(\lambda_2\ddot\beta+2\dot\lambda_2\dot\beta)\vec{y}_2$$

Solution :

1- $\bullet\ \vec{V}(B\in 2/0) = \frac{d}{dt/0}\overrightarrow{AB}$

$= \frac{d}{dt/0}(\lambda_1 \vec{x}_1)$

$= \dot{\lambda}_1 \vec{x}_1 + \lambda_1 \frac{d}{dt/0}\vec{x}_1 \qquad \left(\frac{d\vec{x}_1}{dt/0} = \underbrace{\frac{d\vec{x}_1}{dt/1}}_{=\vec{0}} + \underbrace{\vec{\omega}(1/0)\wedge\vec{x}_1}_{\dot{\alpha}\,\vec{y}_1\wedge\vec{x}_1}\right)$

$\vec{V}(B\in 2/0) = \dot{\lambda}_1 \vec{x}_1 - \lambda_1\dot{\alpha}\,\vec{z}_1$

$\bullet\ \vec{V}(C\in 4/0) = \frac{d}{dt/0}\overrightarrow{AC}$

$= \frac{d}{dt/0}(\overrightarrow{AB} + \overrightarrow{BC})$

$= \underbrace{\frac{d}{dt/0}(\overrightarrow{AB})}_{\text{déjà calculée}} + \frac{d}{dt/0}(\lambda_2 \vec{x}_2)$

$= \_\_\_\_\_\_ + \left(\frac{d}{dt/0}\lambda_2\right)\cdot\vec{x}_2 + \lambda_2\cdot\left(\frac{d}{dt/0}\vec{x}_2\right)$

$= \_\_\_\_\_\_ + \dot{\lambda}_2\vec{x}_2 + \lambda_2\left(\cancel{\frac{d}{dt/2}\vec{x}_2} + \vec{\omega}(4/0)\wedge\vec{x}_2\right)$

$= \_\_\_\_\_\_ + \dot{\lambda}_2\vec{x}_2 + \lambda_2\left(\dot{\alpha}\,\vec{y}_1 + \dot{\beta}\,\vec{z}_2\right)\wedge\vec{x}_2$

$= \_\_\_\_\_\_ + \dot{\lambda}_2\vec{x}_2 - \lambda_2\dot{\alpha}\cos\beta\,\vec{z}_2 + \lambda_2\dot{\beta}\,\vec{y}_2$

$\vec{V}(C\in 4/0) = \dot{\lambda}_1\vec{x}_1 - (\lambda_1\dot{\alpha} + \lambda_2\dot{\alpha}\cos\beta)\vec{z}_2 + \dot{\lambda}_2\vec{x}_2 + \lambda_2\dot{\beta}\,\vec{y}_2$

2- calcule des accélérations:

$-\ \vec{\Gamma}(B\in 2/0) = \frac{d}{dt/0}\vec{V}(B\in 2/0)$

$= \frac{d}{dt/0}\left(\dot{\lambda}_1\vec{x}_1 - \lambda_1\dot{\alpha}\,\vec{z}_1\right)$

$= \frac{d}{dt/0}(\dot{\lambda}_1\vec{x}_1) - \frac{d}{dt/0}(\lambda_1\dot{\alpha}\,\vec{z})$

$= \ddot{\lambda}_1\vec{x}_1 - \dot{\lambda}_1\dot{\alpha}\,\vec{z}_1 - \dot{\lambda}_1\dot{\alpha}\,\vec{z}_1 - \lambda_1\dot{\alpha}^2\vec{x}_1$

$\vec{\Gamma}(B\in 2/0) = (\ddot{\lambda}_1 - \lambda_1\dot{\alpha}^2)\vec{x}_1 - 2\dot{\lambda}_1\dot{\alpha}\,\vec{z}_1.$